"It's Rocket Science!"

Answering the question: How high did my air powered rocket go?

Roger G. Gilbertson
roger@AirBurstRockets.com

I. A LITTLE HISTORY

"In 1666, as tradition has it, [Sir Isaac] Newton observed the fall of an apple in his garden at Woolsthorpe, later recalling, 'In the same year I began to think of gravity extending to the orb of the Moon.' Newton's memory was not accurate. In fact, all evidence suggests that the concept of universal gravitation did not spring full-blown from Newton's head in 1666 but was nearly 20 years in gestation. Ironically, Robert Hooke helped give it life. In November 1679, Hooke initiated an exchange of letters that bore on the question of planetary motion. Although Newton hastily broke off the correspondence, Hooke's letters provided a conceptual link between central attraction and a force falling off with the square of distance."

– From "Sir Isaac Newton" by Dr. Robert A. Hatch - University of Florida
<http://web.clas.ufl.edu/users/rhatch/pages/01-Courses/current-courses/08sr-newton.htm>

Newton and Hooke's key observation about gravity, that the force of gravity decreases in proportion to the second power of the distance, also explains the "up and down" flight of a ballistic object like air powered rockets.

II. SOME NOT-SO-SCARY EQUATIONS

The following equation gives the time t for a falling object to cover a given distance x, in a gravitational field that provides an acceleration force of g.

$$t = \sqrt{\frac{2x}{g}} \tag{1.1}$$

At the surface of the Earth, the acceleration of gravity, g, is equal to 9.8 meters per second squared.

(Notice how no factor in equation 1.1 involves the *mass* of the object falling... this means that all objects fall the same regardless of their mass! In the vacuum on the Moon, a feather and a hammer fall at the same rate. Of course, on Earth air resistance plays a big factor in rocket performance. More about that later.)

Now, if you only know t, the time it took an object to fall, you can calculate the distance it fell by rewriting the equation:

$$x = (t^2 g) / 2 \tag{1.2}$$

With air powered rockets, the maximum speed occurs at the moment the rocket leaves the launcher, since no further force can be applied to it once it departs.

If we consider the rocket to be on a ballistic flight under ideal conditions (i.e. no friction due to the air), the rocket will travel upwards to its peak and return to the ground in equal amounts of time (time up = time down). If you know the total time of flight, t_{total}, then divide it by two to get the time to maximum altitude, and use the equation above to get the maximum altitude:

$$Maximum\ Altitude = ((t_{total} / 2)^2\ g) / 2 \qquad (1.3)$$

III. SOME USEFUL TABLES

If you have the total flight time of your rocket (from launch to hitting the ground), the following table gives a rough estimate of the maximum altitude it much have reached, using equation 1.3.

Acc. of gravity (meter/sec^2)	9.8															
Total flight time (in seconds)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Max Altitude (in meters)	1.2	4.9	11	20	31	44	60	78	99	122	148	176	207	240	276	314
Max Altitude (in feet)	4	16	37	65	102	147	200	261	331	408	494	588	690	800	919	1045

Table 1. Rough estimate of altitude from total flight time.

However, in actual flights, the climb and fall times are not equal. In general, the time to climb to maximum altitude is less than half of the flight time, due to the high initial speed of the rocket, and the time to fall back down to earth is more than half of the flight time, due to the air resistance that limits the maximum speed of the falling rocket (known as the "terminal velocity").

In our tests, performed with actual rockets at sea level at a variety of power levels, we've found an average ratio for climb time versus fall time of around 4:5. The factor f is the percentage of the total flight time used in the climb portion of flight (in this example 4/9 or about 0.44). Adjusting equation 1.3 to include this ratio this gives:

$$Maximum\ Altitude = ((f\ t_{total})^2\ g) / 2 \qquad (1.4)$$

And substituting equation 1.4 into Table 1 gives the following adjusted calculated altitudes in Table 2:

Ratio Climb Time to Fall Time	4 :	5														
Acc. of gravity (meter/sec^2)	9.8															
Total flight time (in seconds)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Max Altitude (in meters)	1.0	3.9	9	15	24	35	47	62	78	97	117	139	164	190	218	248
Max Altitude (in feet)	3	13	29	52	81	116	158	206	261	323	390	465	545	632	726	826

Table 2. Better estimate of altitude by including ratio of rise time to fall time.

So, go fly some rockets, measure the times (use a video camera with a frame counter for real accuracy), then plug in the times to the above equations and see how high they went.

Happy flying!

Roger G.

With thanks to Zach Radding and Mark Forti.